# 12 Conclusion

We have presented a mathematical framework that results in the specification of the broadest possible class of linear stochastic processes. The remarkable aspect is that these continuous-domain processes are either Gaussian or sparse, as a direct consequence of the theory. While the formulation relies on advanced mathematical concepts (distribution theory, functional analysis), the underlying principles are simple and very much in line with the traditional methods of statistical signal processing. The main point is that one can achieve a whole variety of sparsity patterns by combining relatively simple building blocks: non-Gaussian white noise excitations and suitable integral operators. This results in non-Gaussian processes whose properties are compatible with the modern sparsity-based paradigm for signal processing. Yet, the proposed class of models is also backward compatible with the linear theory of signal processing (LMSE = linear mean square estimation) since the correlation structure of the processes remains the same as in the traditional Gaussian case—the processes are ruled by the same stochastic differential equations and it is only the driving terms (innovations) that differ in their level of sparsity. On the theoretical front, we have highlighted the crucial role of the generalized B-spline function $\beta_{\mathrm{L}}$—in one-to-one relation with the whitening operator L—that provides the functional link between the continuous-domain specification of the stochastic model and the discrete-domain handling of the sample values. We have also shown that these processes admit a sparse wavelet decomposition whenever the wavelet is matched to the whitening operator.

Possible applications and directions of future research include:

- The generation of sparse stochastic processes. These may be useful for testing algorithms or for providing artificial sounds or textures.
- The development of identification procedures for estimating: 1) the whitening operator L and 2) the noise parameters (Lévy exponent). The former problem may be addressed by suitable adaptation of well-established Gaussian estimation techniques. The second task is less standard and potentially more challenging. One possibility is to estimate the noise parameters in the transformed domain (generalized increments or wavelet coefficients) using cumulants and higher-order statistical methods.
- The design of optimal denoising and restoration procedures that extend upon the first-order techniques described in Sections 10.4 and 11.4. The challenge is to develop efficient algorithms for MMSE signal estimation using the present class of

sparse prior models. This should provide a principled approach for tackling sparse signal recovery problems and possibly result in higher quality reconstructions.

- The formulation of a model-based approach for regularization and wavelet-domain processing with the derivation of optimal thresholding estimators.
- The definition and investigation of richer classes of sparse processes—especially in higher dimensions—by mixture of elementary ones associated with individual operators $\mathrm{L}_i$.
- The theory predicts that, for the proposed class of models, the transformed domain statistics should be infinitely divisible. This needs to be tested on real signals and images. One also needs to design appropriate estimators for capturing the tail behavior of the pdf which is the most important indicator of sparsity.